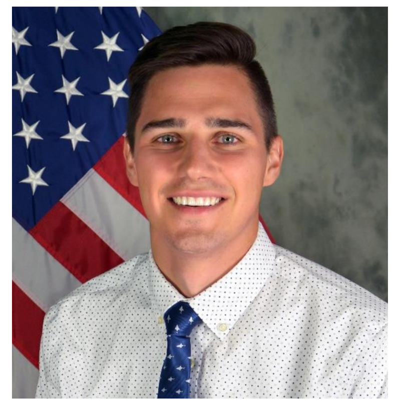
Nonlinear substructure methods to efficiently predict mechanical responses



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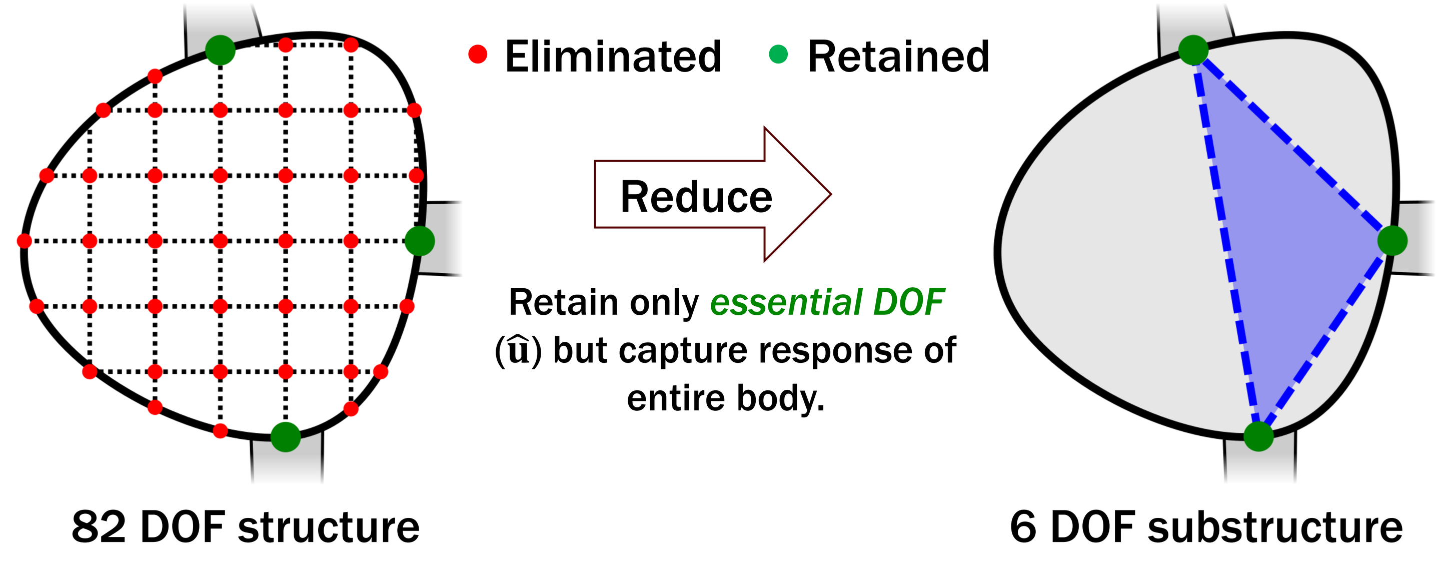
Background:

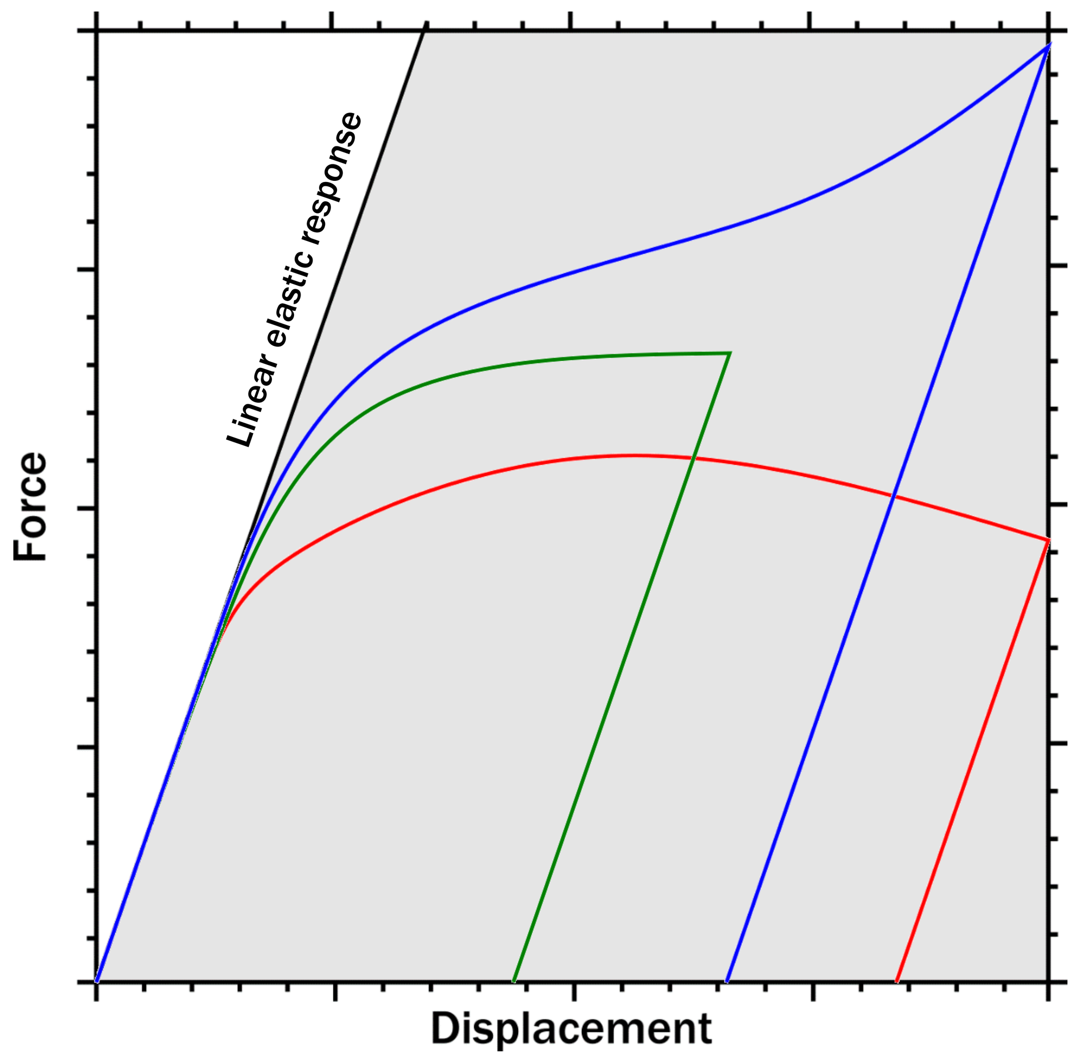
Substructure analysis reduces the computational cost associated with predicting the response of a body by eliminating non-essential degrees of freedom (DOF) [1].

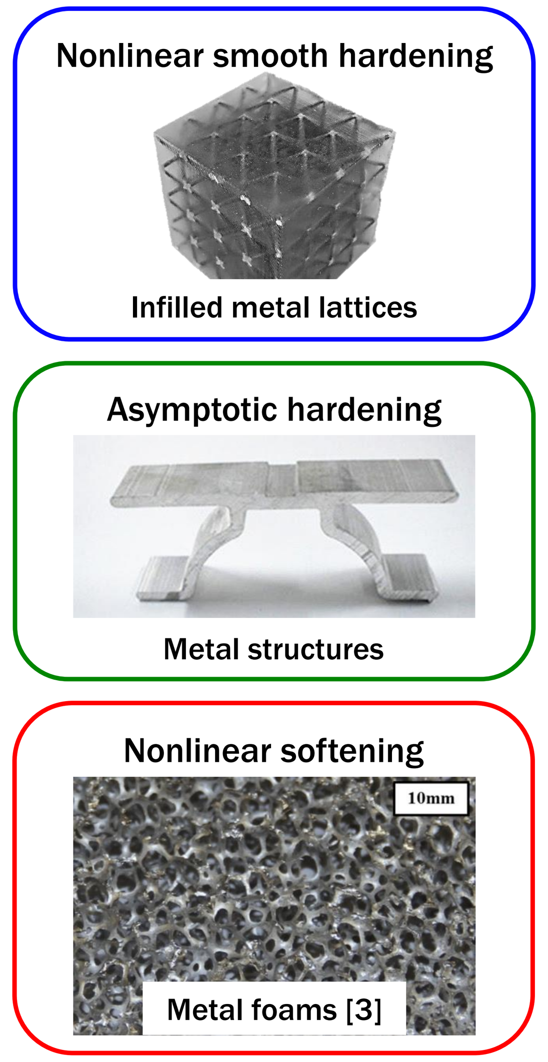


Notional depiction of substructure analysis, in which a meshed structure is reduced from 82 degrees of freedom to a 6 degree of freedom substructure.

Objective:

Leveraging the mathematical framework developed for constitutive plasticity, develop a scheme to account for *general nonlinear structural responses of arbitrary order* [2].





The mathematics of constitutive plasticity can capture a wide range of general nonlinear responses, including material yield, hyperelasticity, and large deformations.

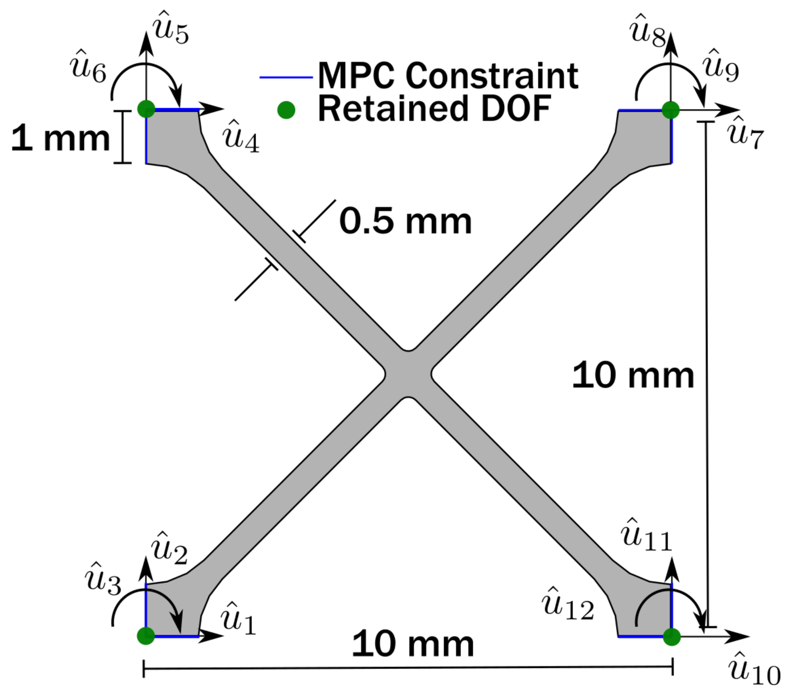
Framework components:

- Deformation decomposition (defined by the known *linear substructure solution)*
- Nonlinear initiation criteria
- Evolution equations that govern how internal state variables evolve

Example: Lattice truss with nonlinear smooth hardening constitutive nonlinearity

The nonlinear substructure workflow of reduction, training, and calibration is accomplished via generation of high-fidelity FEA data and subsequent calibration performed via optimization.

Step 1: Reduction and substructure model formulation



Deformation decomposition

$$\hat{\mathbf{K}}(\hat{\mathbf{u}} - \hat{\mathbf{G}}^{NL}) = \hat{\mathbf{F}}$$

Evolution equations

$$\dot{\hat{\mathbf{G}}}^{NL} = \gamma \frac{\hat{\mathbf{A}}\hat{\mathbf{F}}}{\sqrt{\hat{\mathbf{F}} : \hat{\mathbf{A}}\hat{\mathbf{F}}}}, \ \dot{\alpha} = \gamma$$

Nonlinear initiation function

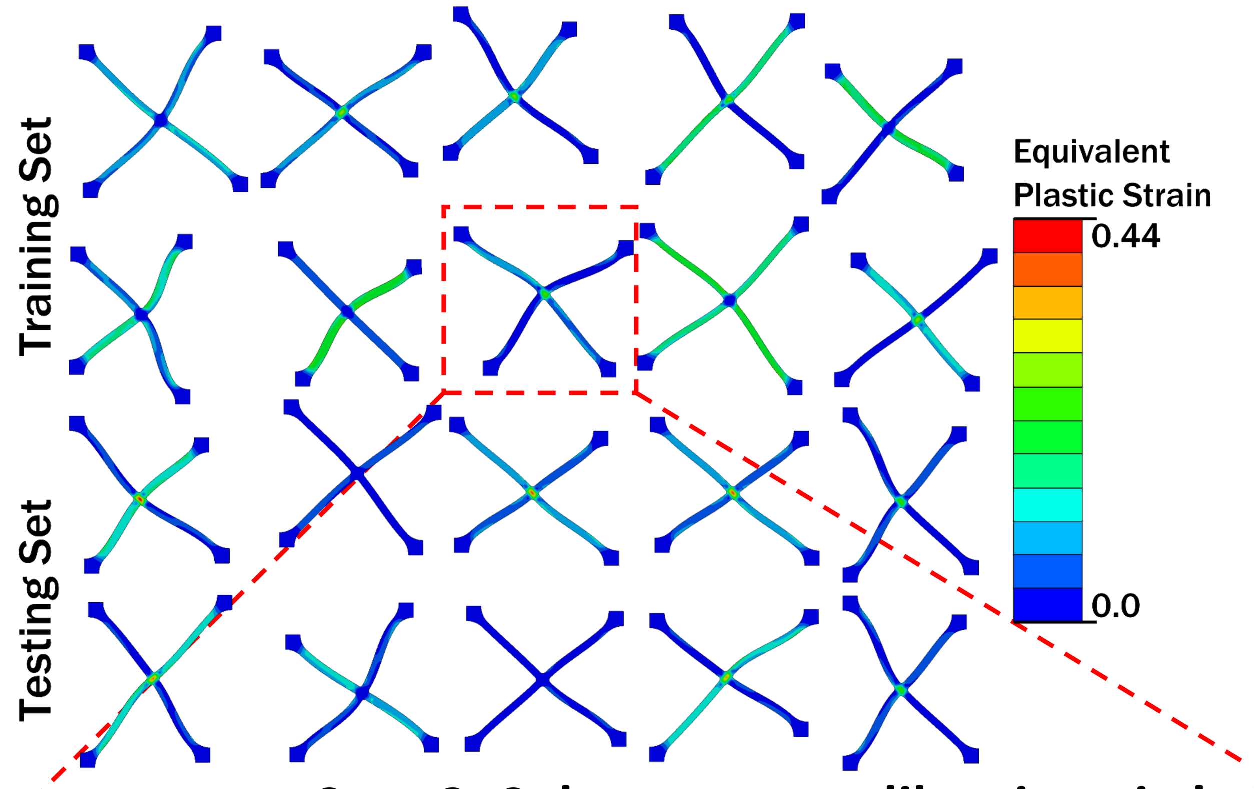
$$f = \tilde{f}(\hat{\mathbf{F}}) - \check{f}(\alpha)$$

$$\tilde{f}(\hat{\mathbf{F}}) = \sqrt{\hat{\mathbf{F}} : \hat{\mathbf{A}}\hat{\mathbf{F}}}$$

$$\check{f}(\alpha) = F_0^y + \frac{1}{2}M(\xi + \alpha^{n_1} - (\xi - \alpha)^{n_2})$$

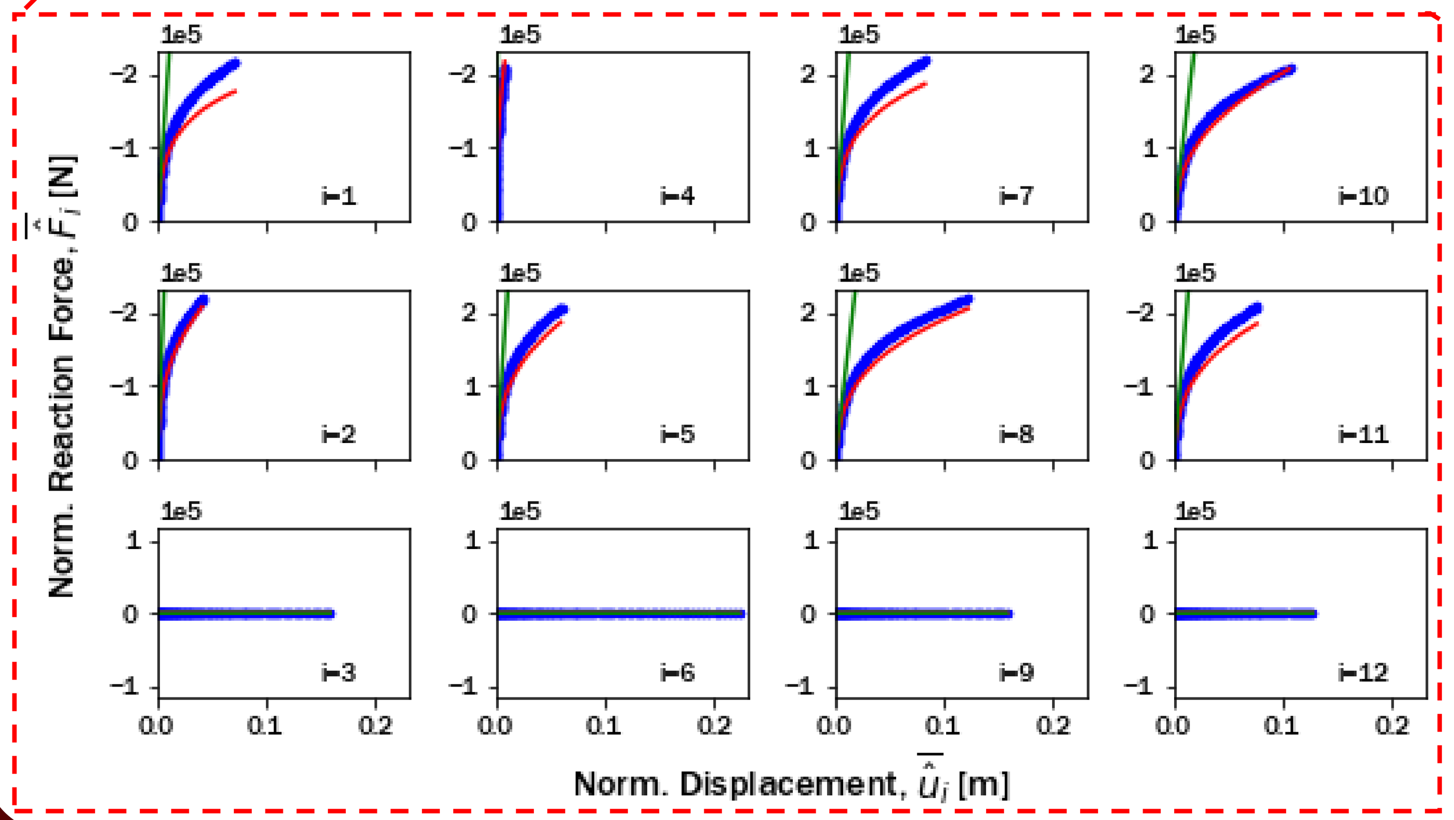
***Red** = substructure model parameter (found via calibration)*

Step 2: Generation of training and testing data via Latin Hypercube Sampling



Equivalent plastic strain contours of the training and testing load cases in high-fidelity FEA. Time histories of force-displacement (or moment-rotation) pairs for all retained degrees of freedom are recorded. In this example, 10 load cases are used for training and testing.

Step 3: Substructure calibration via hybrid optimization



Agreement between substructure prediction and training data for a selected load case.

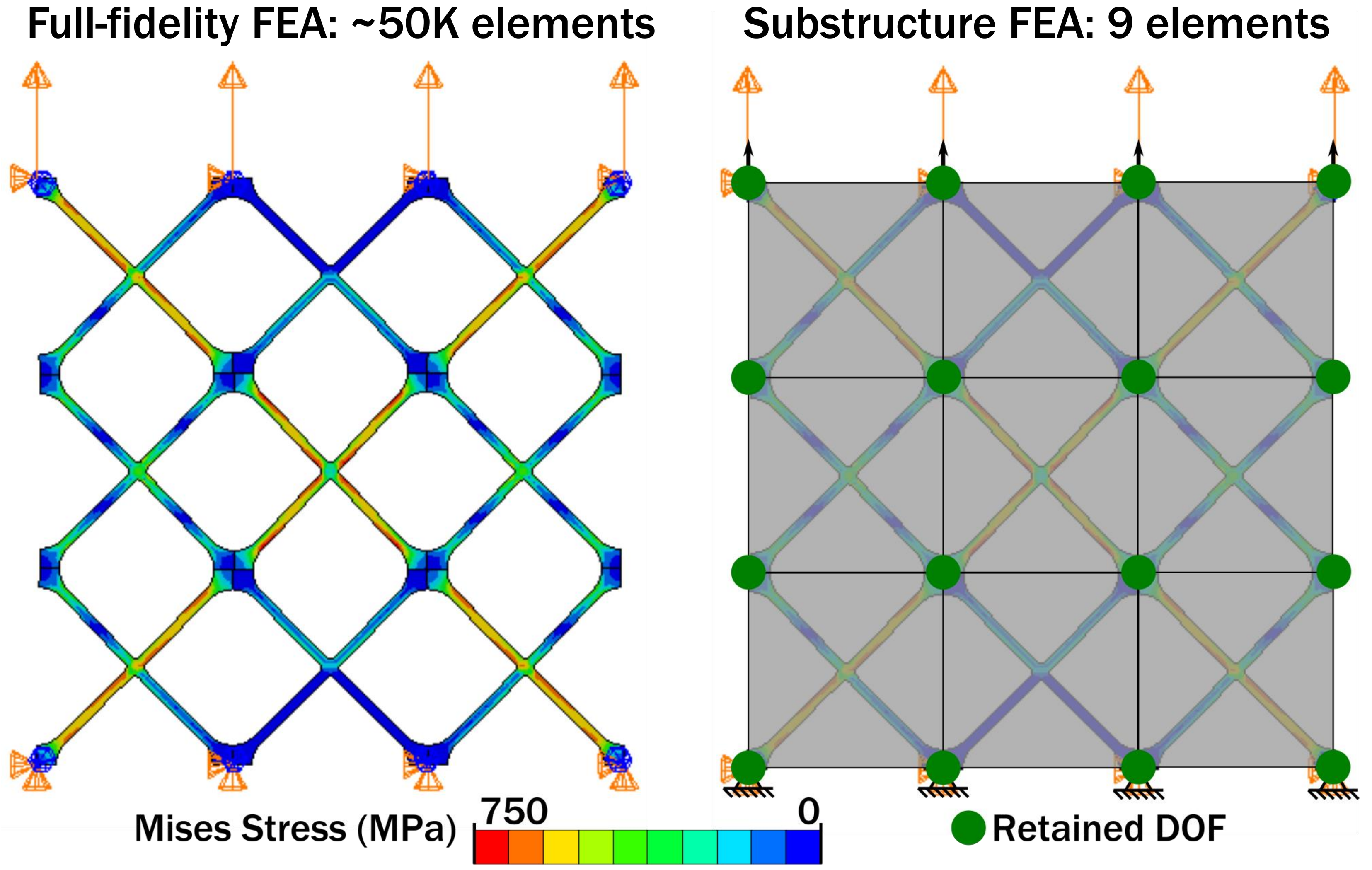
Calibration error metrics:
Training Error: 166.5
Testing Error: 159.4

Conclusions:

- The mathematics developed for constitutive plasticity can be extended to apply to higher-dimensional structural bodies.
- General nonlinear responses involving complex structures can be predicted by the aforementioned framework at a fraction of the computational cost of traditional FEA.

Future work:

- Integrate the nonlinear substructure process to predict the response of multiple unit cells assembled together.
- Perform multiscale optimization considering heterogeneous configurations and multiple different types of unit cell geometries.



Comparison between traditional full-fidelity FEA and the substructure analog for a 3-by-3 array of lattice structures. Substructures could provide immense speedup by only requiring 9 functional evaluations per loading increment, compared to the 50,000 functional evaluations required in traditional FEA.

Acknowledgments:
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References:

[1] J. S. Przemieniecki, *Matrix Structural Analysis of Substructures*, AIAA J., vol. 1, no. 1, pp. 138–147, Jan. 1963, doi: 10.2514/3.1483

[2] Simo, Juan C., and Thomas JR Hughes. *Computational inelasticity*. Vol. 7. Springer Science & Business Media, 2006.

[3] S. Chen, *A Study on Properties of Novel Metallic Foam for Nuclear Applications*. North Carolina State University, 2015.